Evaluating Probability of Collision Uncertainty

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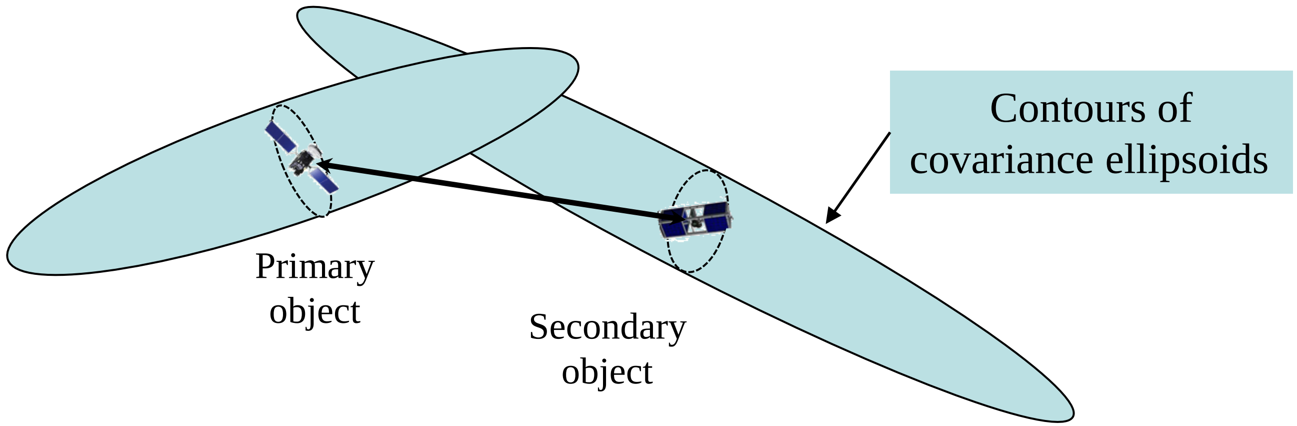
Background: Conjunction Assessment

- **Conjunction Assessment Risk Analysis (CARA)**
  - Evaluate collision risk between two conjuncting objects
  - Mitigate collision risk, if necessary



- **Probability of Collision (Pc) is a single-parameter encapsulation of the risk and is computed from**
  - Miss distance at time of closest approach (TCA)
  - State estimation error (covariance) for both objects
  - Hard-body radius (HBR) of both objects

2-D Pc Computation

- **Define plane perpendicular to velocity vector ("conjunction plane")**
  - If a collision will occur, it will occur in this plane
- **Combine primary and secondary covariances**
- **Project combined covariance into conjunction plane, at origin**
- **Place primary location one miss distance away, on x axis**
- **HBR is defined as circle (with appropriate area) placed at that point**
- **Pc is then the portion of the density that falls within the HBR circle**
  - r is [x z] and C* is the projected covariance

$$P_C = \frac{1}{\sqrt{(2\pi)^2 |C^*|}} \iint_A \exp\left(-\frac{1}{2}\vec{r}^{\,T} C^{*-1} \vec{r}\right) dXdZ$$

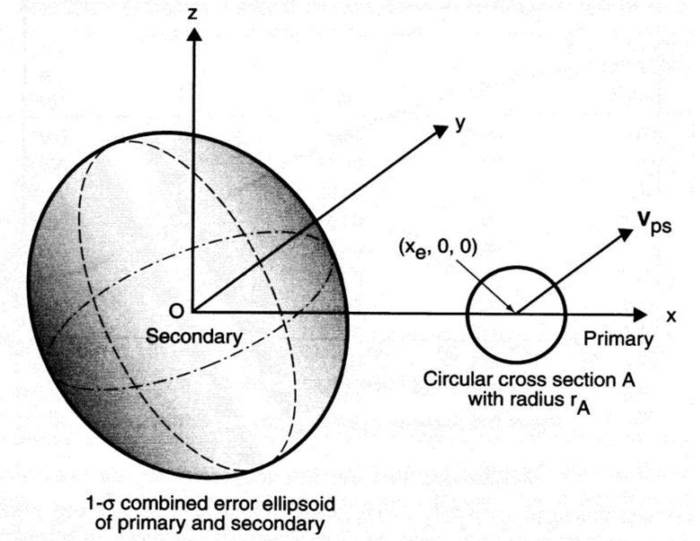


Figure taken from Chan (2008)

Probability of Collision Calculation

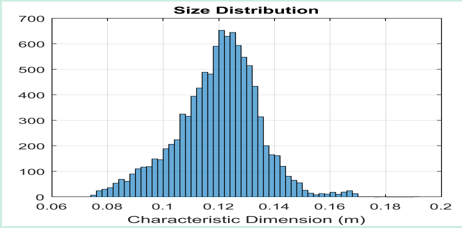
- **Pc is only a nominal solution for the conjunction**
  - Derived from estimates of the mean
    - If error distributions non-Gaussian, then this is not an expression of central tendency
  - Does not include uncertainties on the inputs
    - "Uncertainty of uncertainty volumes" or HBR
- **Thus, while representing the risk, nominal Pc is just a point estimate**
- **Want to know how much variation or uncertainty in the Pc calculated for any given conjunction**
  - Determine uncertainty PDFs for the Pc calculation inputs
  - Through Monte Carlo trials, vary above inputs to the Pc calculation
  - Include a resampling technique to determine natural variation
  - Generate a probability density of resultant Pc values
  - Characterize this distribution empirically

Uncertainty in the Probability

- **Generate a Pc distribution, using Monte Carlo (MC) trials of the underlying uncertainties**
  - Determine uncertainty for each of the Pc parameters

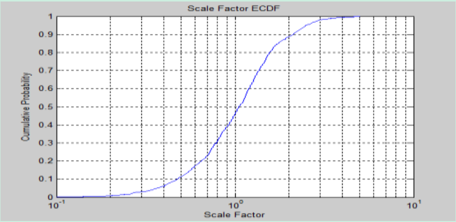
Underlying Uncertainties

HBR Uncertainty



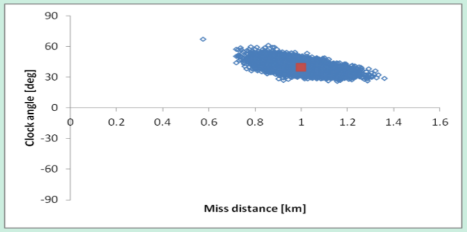
Draw from projected area distribution (primary) and RCS PDF (secondary)

Covariance Uncertainty



Draw from scale factor distributions for both objects

Natural Sampling Variability

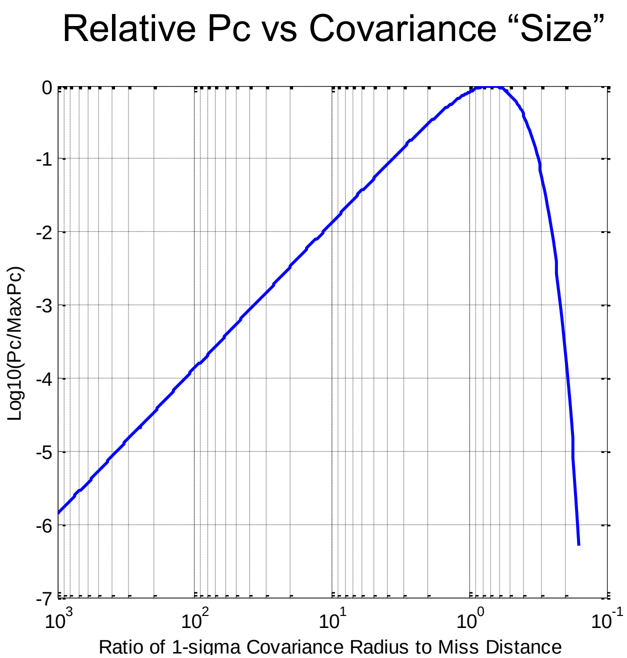


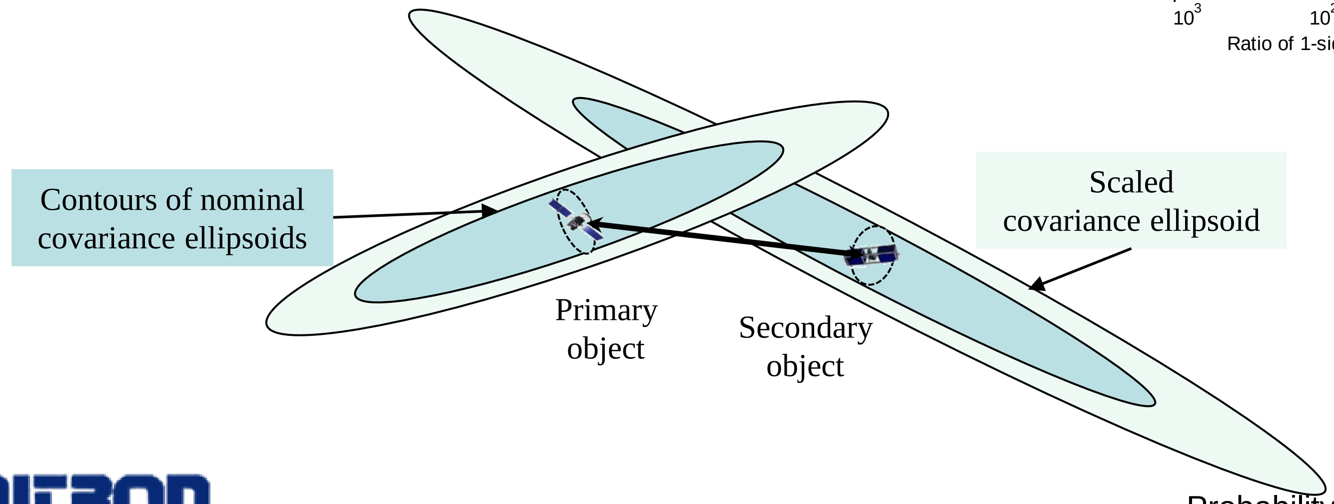
Draw from 2D scaled Gaussian covariance

Generate P_c distribution

Covariance Uncertainty

- **Changes in covariance sizes can change calculated Pc, sometimes substantially**
  - Especially if on right side of canonical curve
- **Need to know range of values for appropriate scale factors for covariances**
  - Typical applied range is from 0.2 to 5, but this is unrealistically large for nearly all cases
  - Should be object-specific
  - Should include probabilistic element



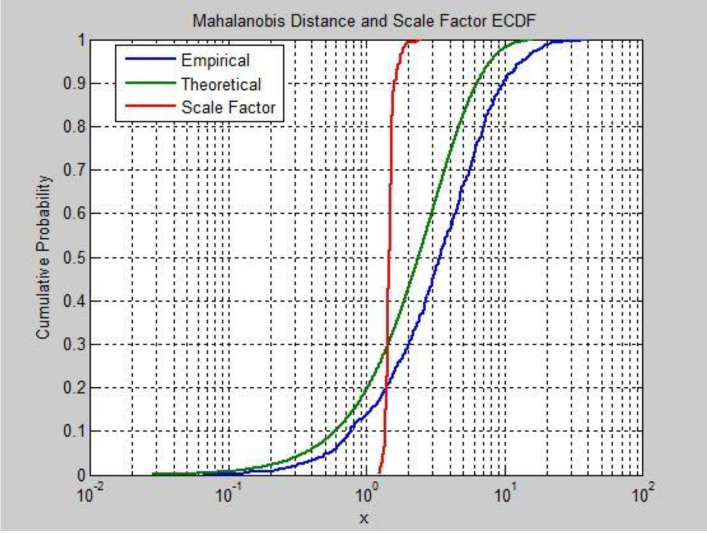


Covariance Uncertainty: Evaluation Products

- **JSpOC-resident utility generates reference orbits for every satellite**
  - Similar methodology as that used for SLR precision ephemerides
  - Covariance data from generating ODs preserved
- **Second utility compares each generated SP vector to reference orbit at propagation points of interest**
  - 1, 2, 3, 5, and 7 days from epoch
  - Calculates position residuals and combined covariance, which is combination of propagated vector covariance and reference orbit covariance
- **With position residuals and combined covariance, can compute covariance "realism" factor for each vector at each prop point**
  - For each vector, can calculate $\varepsilon C^{-1}\varepsilon^T$ (M^2, square of Mahalanobis distance)
    - ε is the vector of position residuals; C is the combined covariance
  - If covariance realistic, M^2 set should produce a 3-DoF chi-squared distribution

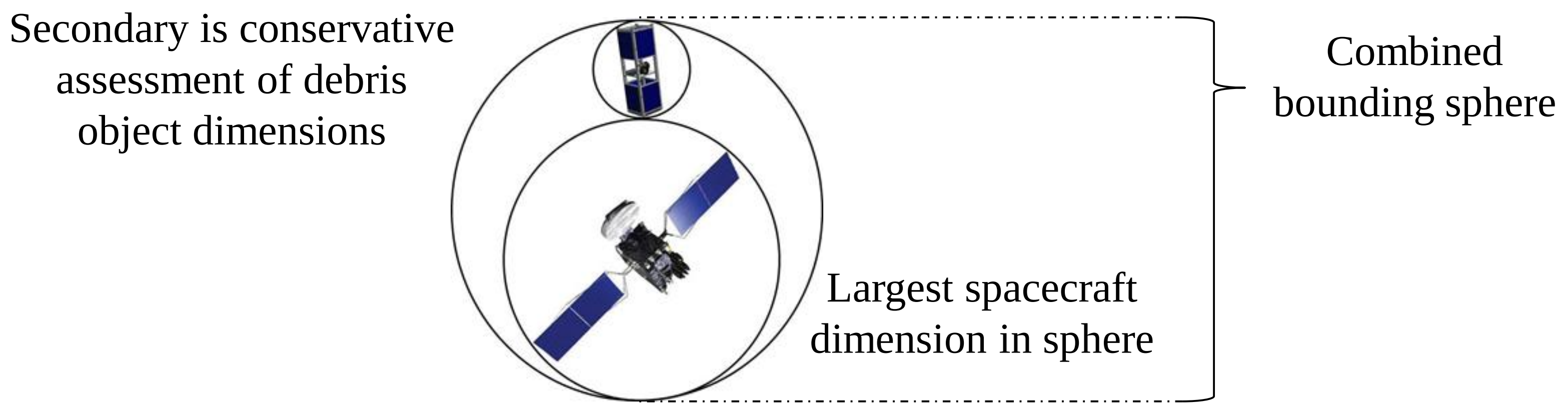
Mahalanobis Distances to Scale Factors

- **Presume set of 100 M2 factors generated for a satellite**
  - Rank-order the 100 factors
  - Align each with the 3-DoF chi-squared value for that given percentile
    - E.g., factor #20 aligned with 20th percentile chi-squared value
  - Empirical / ideal value is scale factor for each instance
    - Value by which covariance would need to be multiplied to produce ideal chi-squared value for that percentile point
  - Set of 100 scale factors now available for Monte Carlo draws
- **Sets of these calculated for every satellite for propagation points of interest**



Hard-Body Radius

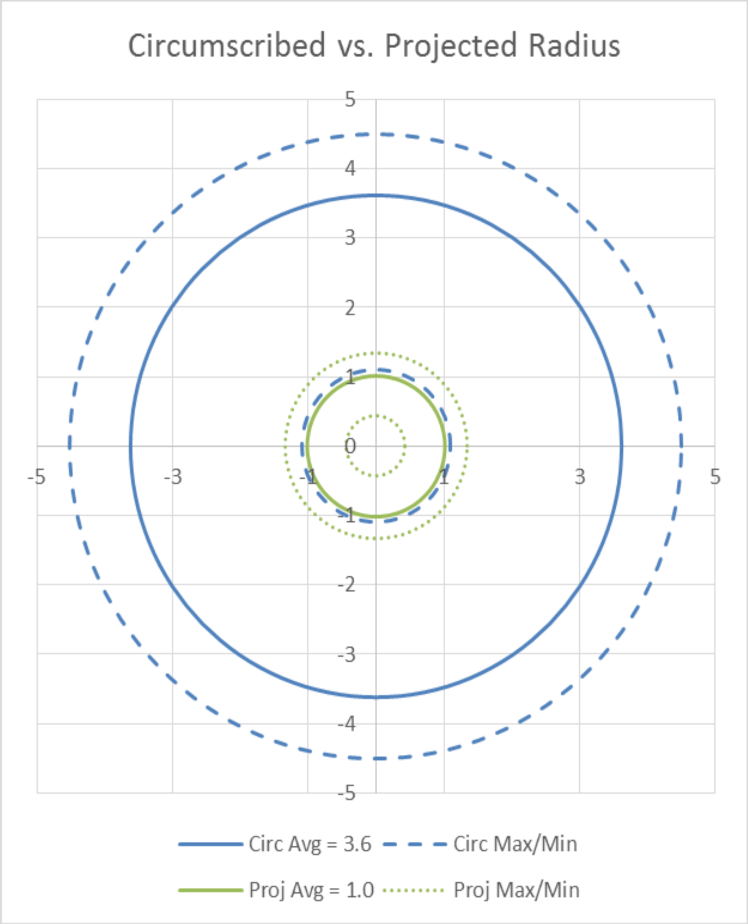
- **HBR is typically found by circumscribing both objects in spheres and combining the objects into one bounding sphere**
  - Size of the secondary is typically not known, so added as a large estimate of debris object dimensions



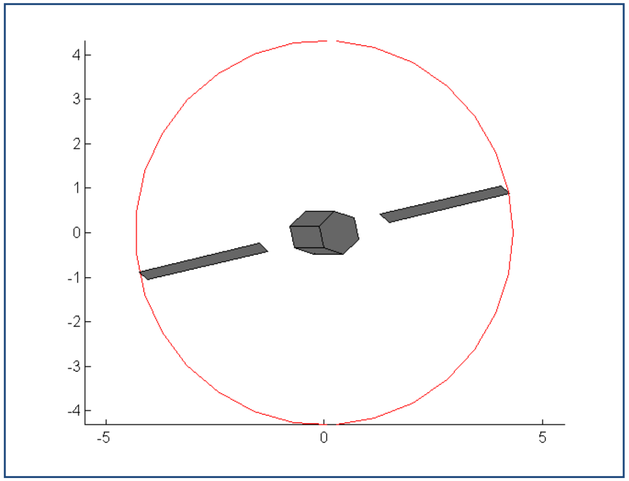
- **HBR uncertainties that follow represent a more realistic estimate of the area in the conjunction plane**
  - The combined uncertainties are much smaller than the bounding sphere

Primary Object HBR Uncertainty: PDF of More Realistic Values

- **Uncertainty estimated by the projected area of the spacecraft in a random orientation on the conjunction plane**
  - Simplified geometric model of the spacecraft
  - Save the projection areas to a PDF
    - Projected area expressed as a circular radius



Geometric model of OCO-2 in arbitrary orientation on conjunction plane



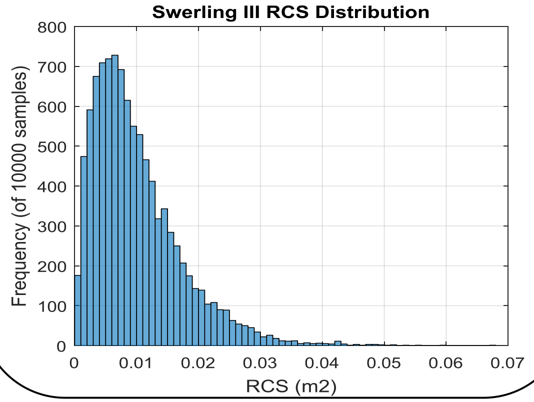
Secondary Object HBR Uncertainty (1 of 2)

- **For intact spacecraft, possible to use published dimensions**
  - For payloads, these are often not precise enough to be useful, and at least some canonical models would have to be imposed
    - Error in all of this great enough that approach is questionable
  - For rocket bodies, published dimensions are probably adequate
    - But many booster types lack published dimensions
- **Most common secondaries are debris objects, for which no size information is available**
- **Can try to estimate size from RCS value**
  - CDFs of individual objects' RCS values not available, so must assume canonical distribution
    - 2010 study showed Swerling III to be most common for debris; also most conservative in terms of object sizes
    - Can scale this distribution by average RCS value in CDM to size it properly for any particular debris object
  - Then must transform RCS values to size values

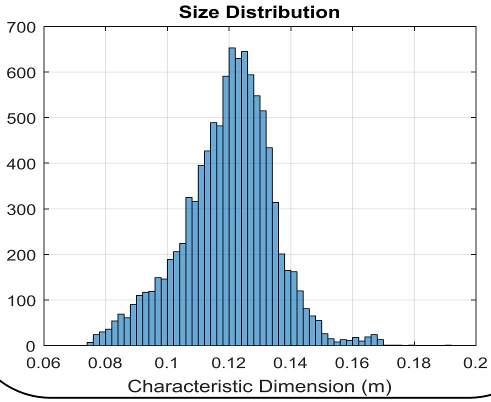
Secondary Object HBR Uncertainty (2 of 2)

- **Converting RCS to physical size**
  - Can assume object to be a perfectly conducting sphere
    - Not only a bad assumption for debris, but renders non-unique size solutions
  - Can use ODPO-developed Size Estimation Model
    - Certified only for debris smaller than 20cm and then only to convert an entire PDF of RCS values to a PDF of characteristic dimensions
    - Somewhat off-label use, but true to restriction of converting PDFs of data rather than single values

Generate RCS distribution from properly-located Swerling III model



Convert to a size distribution using ODPO Size Estimation Model



Pc Calculation Resampling

- **Resampling/bootstrap methods often used to generate confidence intervals when calculation final distribution unknown**
- **Early attempts at this with Pc used resampling with invariant covariances**
  - Take position draw on primary and secondary covariance at TCA
  - Find new TCA; this defines new nominal miss vector
  - Recompute Pc with this new miss vector and unaltered covariances
  - Problem: covariance is clearly correlated with miss distance
    - Cannot produce new miss distance from covariance-based sampling and then recompute Pc using those same covariances
- **Need approach that considers miss distance / covariance linkage**

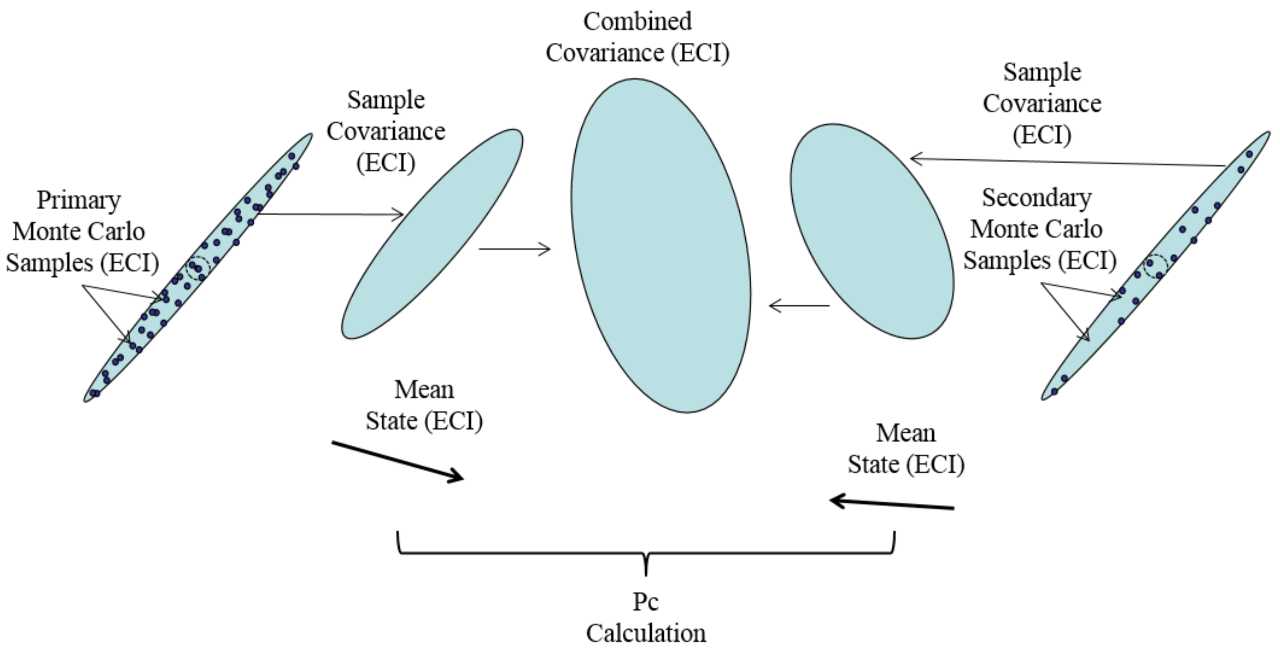
Pc Calculation Resampling Proposed Approach

- **J.H. Frisbee proposed a resampling technique that would also address the correlation problem**
  - Choose samples from the combined covariance to generate *m* miss distances
  - Take mean of *m* miss distances—this is new nominal miss distance
  - Take sample covariance of *m* miss distances—this is new combined covariance
  - Compute Pc from this mean miss distance and sample combined covariance
  - Repeat procedure *n* times—this produces bootstrap dataset
- **In this framework, covariances are considered representatives of parent distributions, here further characterized by resampling**
- **Issue: what should be the value of *m*?**
  - In bootstrapping, want the bootstrap sample size to equal the single-sample size that would have been used (or was used) to estimate the parameter
  - Thus, want the number of samples (DoF) of the bootstrap resampling (*m*) to equal the DoF that produced the covariance in the first place
    - That is, the DoF of the generating OD

Tracking Levels and Degrees of Freedom

- **DoF is usually calculated as the number of data points minus the number of estimated parameters**
  - JSpOC ODs calculated with SSN obs (usually have range, azimuth, and elevation—three observables)
  - Obs provided in “tracks”—group of obs taken during one tracking session
- **Thus, tabulation issues arise**
  - Each ob provides 3 DoF, minus the estimated parameters
  - However, rather little information content in interior obs of a track
    - JSpOC “track weighting” confirms this—all tracks weighted the same in the OD, regardless of length
  - Better tabulation to count each track as equivalent of one state estimate
    - Longish track about enough data to execute a single state estimate
    - Total estimated parameters in OD would thus be only one—one state estimated
  - DoF calculation is thus “# of tracks – 1”
    - Would need to be amended for DS, where obs report only two parameters

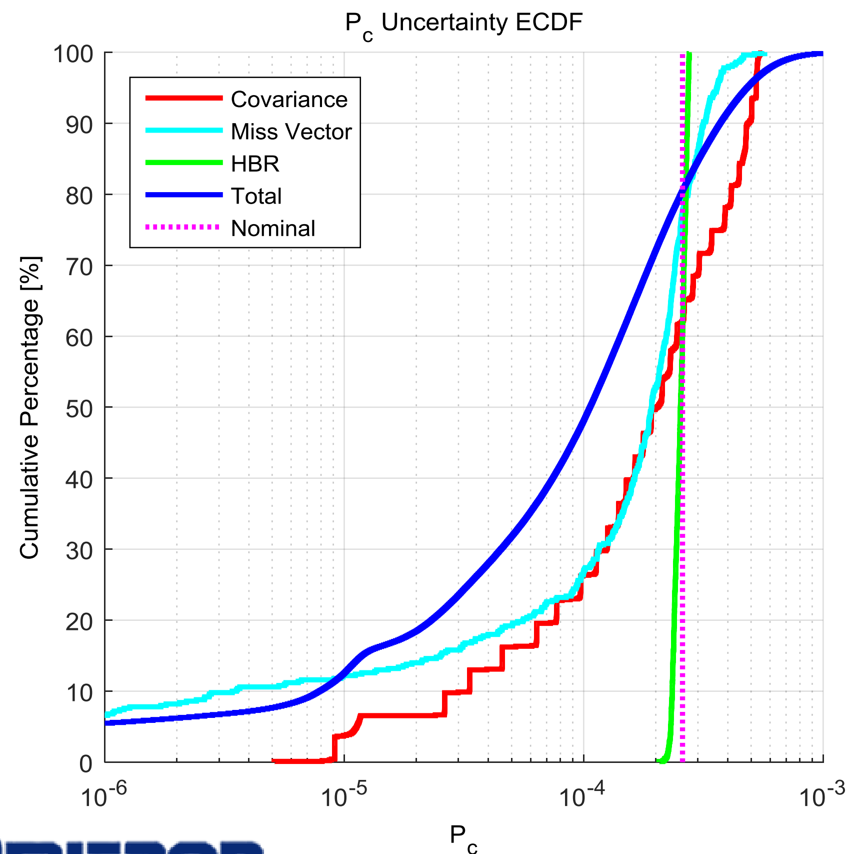
Resampling Approach Schematic

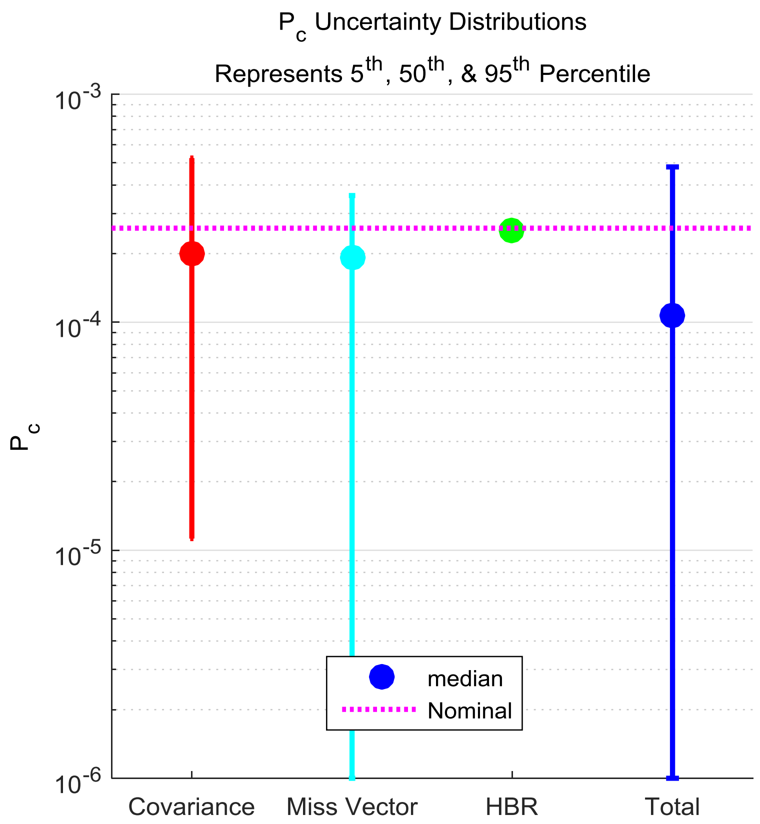


- **Repeated thousands of times to calculate distribution of Pc values**
- **Benefits**
  - Correlation of the miss vector and the covariance
  - Maintains an equivalent sampling level to the original OD
    - Naturally responds to variations in tracking density

Pc Uncertainty Plot

- **Each perturbation (covariance realism, HBR, and resampling) is plotted without inputs from the other perturbations**
- **Total (blue) line combines all the perturbations**





Uncertainty Plot Interpretation

- **Fixed primary size against debris: very little HBR variation**
- **Covariance scaling and natural variation have wider spreads; in each case median lies below nominal value**
- **Total line has 80% of points below nominal value**
- **CARA usual threshold for remediation is ~4E-04**
  - If worried that nominal is close to this and therefore remediation should perhaps be considered, fewer than 5% of points over that value
    - So can dismiss that possibility fairly easily
- **Some users set 1E-04 as remediation threshold**
  - Right at median level for Total line

Conclusions and Future Work

- **Proposed method**
  - Characterizes the PDF that can represent the Pc from a particular conjunction, given the uncertainties in covariances, HBR and natural variation in the Pc calculation
  - Gives a sense of the dynamic range of the Pc and allow maneuver decisions to be based on percentile points of this range rather than the nominal value alone
  - Provides a mechanism for obtaining a better expression of the calculation's central tendency (here the median)
- **Future Work**
  - Refine DoF calculation and generate expansion for angles-only cases
  - Survey results from runs of large datasets
    - Stability studies of simplifying assumptions for faster processing
  - Examine potential as a Pc forecaster